ISSN 2229-5518

Spectrum Sensing using Compressed Sensing Techniques for Sparse Multiband Signals

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Abstract— Spectrum is scarce and the primary users (licensed users) do not use them always. There are free spaces called spectrum holes. Spectrum is not utilised efficiently in certain bands. A technique which scans the spectrum for the given bandwidth and finds the spectrum holes so that secondary users can use them, was proposed. But for high bandwidths the sampling rates are high such that practical Analog to Digital Converters cannot achieve. Compressed sensing techniques sample at rate less than the Nyquist rate and still are able to reconstruct the original signal except that the signal should be sparse in some domain. So spectrum sensing using compressed sensing methods were proposed and found to be more efficient.

Index Terms— Blind spectrum sensing, Cognitive Radio, Compressed Sensing, Randomness, Sparse multiband signals, Spectrum Sensing, Support.

1 Introduction

SPECTRUM is a potentially scarce resource. Spectrum sensing is a process which ensures that Cognitive Radios will not interfere with Primary Users. They reveal the unused bands so that unlicensed users (secondary users) can establish communication in free bands. They are called spectrum holes. As we go for higher frequency applications the sampling frequency will be higher and the practical ADCs impose bandwidth restriction on the signal thus resulting in loss of information.

In order to overcome this problem a method called Compressed Sensing [1] was introduced, which is capable of reconstructing the signal using lesser samples than Nyquist rate. The condition to be satisfied by the signal is that it should be sparse in any domain. The reconstruction method is considered as an optimization problem and can be solved using different algorithms like Orthogonal Matching Pursuit (OMP), Basis Pursuit (BP) etc.,

The available Spectrum Sensing techniques are listed in Section 2.Compressed Sensing based Spectrum sensing techniques [11] are described in Section 3. The Modulated Wideband Converter technique is detailed in Section 4, and the reconstruction algorithms along with results are explained in Section 55, the paper is concluded in Section 6 and Future scope is discussed in Section 7.

2 Spectrum Sensing Techniques

Spectrum sensing techniques perform sensing directly at Nyquist rate. They manipulate the different properties of the signal in

1. Matched Filter Detection
2. Energy Detection
3. Cyclostationary Method

3 Compressed Sensing Based Spectrum Sensing Techniques

In high frequency applications the sampling rate becomes very high. The currently available ADCs are incapable of handling such high rates. So, we employ compressed sampling techniques to obtain the high frequency information at a rate lower than Nyquist rate. Three methods will be discussed below.

1. Random Demodulator
2. Multi- Coset sampling
3. Modulated Wideband Converter

3.1 Random Demodulator

Random demodulator is used to acquire sparse, band limited signals. The input signal is multiplied by a pseudo random sequence which spreads the tone across the entire spectrum. Then a low pass filter is used to capture a part of message in the baseband which is later sampled at a lower rate. The obtained samples do not have linear relationship with the message.

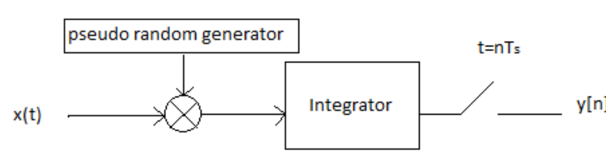

Fig 1. Block diagram of Random Demodulator

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ISSN 2229-5518

Convex optimization teniques are employed for efficient reconstruction of the message. It was found that this method is more efficient for multi tone signals.

3.1 Multi Coset Sampling

Multi coset sampling method is used for compressive sampling of sparse multi band signals.

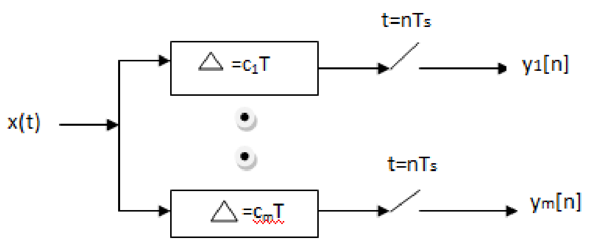

Fig 2. Block diagram of Multi Coset Sampler

The signal is fed into m parallel shifters which are also called cosets maintained at different shifts. These cosets pick up at least one sample from each active band. The effective sampling rate becomes $\frac{m}{MT}$ which is a rate lower than Nyquist rate. So, we obtain different versions of the same signal [10]. ADCs introduce a bandwidth limitation which causes distortion and maintaining time shifts of the order of Nyquist rate is difficult.

4 Modulated Wideband Converter

In this method same signal $x(t)$ is passed parallely through m channels. Each channel has a mixing signal $P_i(t)$ that alias the input signal to baseband. The primary aim of this method is to recover a sparse signal using relatively lesser number of samples. An analog mixer aliases the spectrum, such that a spectrum portion from each band appears in baseband. The system consists of several channels, implementing different mixtures, so that, in principle, a sufficiently large number of mixtures allow to recover a relatively sparse multiband signal.

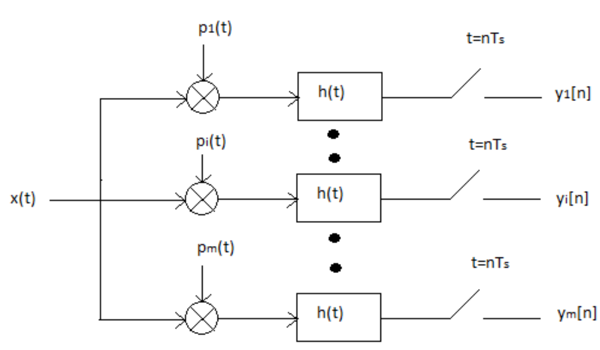

Fig 3. Block diagram of Modulated Wideband Converter

We assume the input signal is a sparse multiband signal $x(t)$ which consists of N active bands each with bandwidth B. We also assume that the signal is sparse in frequency domain. And let the Nyquist frequency of the signal be f_{NYQ} .The signal enters m channels simultaneously. In the i^{th} channel the signal is multiplied by a mixing function, which has a period T_P. After mixing, the signal spectrum is truncated by a low-pass filter with cutoff $\frac{1}{2T_S}$ and the filtered signal is sampled at rate T_S. The sampling rate of each channel is low, so that existing commercial ADCs can be used. Therefore the design parameters are number of channels 'm', the time period T_P , and the sampling rate of the ADC $\frac{1}{T_S}$ and the mixing function $P_i(t)$ for $1 \le i \le m$, where m is the number of channels.

In practice $P_i(t)$ can be any periodic function but here we take it as piecewise constant function alternating between the levels ± 1 for each of M intervals of the time.

$$P_i(t) = \alpha_{ik} \;,\; \frac{\mathrm{k}}{M} \le t \le \frac{(\mathrm{k}+1)\mathrm{T}_p}{M}, \quad 0 \le k \le M-1$$

Where $\alpha_{ik} = \{-1, +1\}$ and $P_i(t + nT_P) = P_i(t)$ shows that $P_i(t)$ is T_P periodic.

If $X(f)$ is the spectrum of the signal and $Y_i[n]$ are the DFTS of each channel, then the relation between them can be shown by the following mathematical equation,

$$Y(f) = AZ(f) \qquad (1)$$

where $Y(f)$ is a vector of length m with ith element being $Y_i(f)$ and $Z(f)$ is the unknown vector to be calculated. This $Z(f)$ gives the spectral support of the input signal. A is an $m \times L$ Matrix and is called the sensing matrix

The necessary conditions for perfect reconstruction of the signal are $f_P = B, f_S \ge f_P$ and $m \ge 2N$

The sufficient conditions for perfect reconstruction of the signal is $f_S \ge f_P \ge B, M \ge M_{\min}$ where $M_{\min} = 2\left\lceil \frac{(f_{NYQ} + f_S)}{2 f_P} \right\rceil - 1,$ and $m \ge 2N$ for blind reconstruction. The relation (1) is the compressed sensing framework and the reconstruction involves complex algorithms for perfect reconstruction of the original [1]. The

ISSN 2229-5518

above relation (1) is called the Infinite Measurement Vector problem.

In the simulation following specifications were used :

$$M_{\min} = 267, L = 267, B = 6Mhz, N = 6,$$
$$m = 100,\ f_S = f_P = 7.5\text{Mhz}.$$

5 RECONSTRUCTION TECHNIQUES WITH RESULTS

5.1 INFINITE MEASUREMENT VECTOR (IMV)

The main aim is to recover the unknown vector set x from the known measurements. It is called an Infinite measurement Vector problem if the set Λ is continuous in time or it consists of large number of samples. The number of unknowns is large in number so that the computation load increases. So another model is proposed.

5.2 MULTIPLE MEASUREMENT VECTOR (MMV)

This model is similar to that of the IMV model except that the cardinality of Λ is finite value 'l'. It is proved that for every IMV model there exists a MMV model such that the support is recovered. In this model we can see that the number of unknowns have been decreased to some extent. But still this is a convex optimization problem so the computation load is high.

5.3 SINGLE MEASUREMENT VECTOR (SMV)

This model is obtained by further reducing the dimension of MMV model. Since we assume that the input bandsparse signal has joint sparsity prior we will be able to breakdown the MMV model into multiple SMV models. As we are dealing with only one vector at a time the computational load decreases significantly.

The reconstruction of the signal from the obtained compressed samples is further split up into two sub problems. The first one is to recover the support $S = I(x(\Lambda))$ from the equation and the second one is to reconstruct the X with the knowledge of S and Y [4].
This can be done in two ways:

1. Recovery of support directly from MMV system using OMP algorithm.
2. Convert the MMV to SMV system and then recovery of support using OMP algorithm.

5.3.1 DIRECT RECOVERY FROM MMV SYSTEM

The OMP algorithm is used to recover the support in this model.

Parameter: Acceptable error E_S.

Algorithm for the equation $Y = AX$:

1.Initialize a vector $A' = \{\}$.
2.Find the column of A that has maximum correlation with Y.
3.Add that column to A'.
4.Perform reconstruction using A'.
5.Calculate mean square error E.
6.If $E \le E_S$ then A' is the reduced matrix of A and it is enough to reconstruct X from Y.
7.Else remove the maximum correlated column from A and repeat from step 2.
8.Stop.

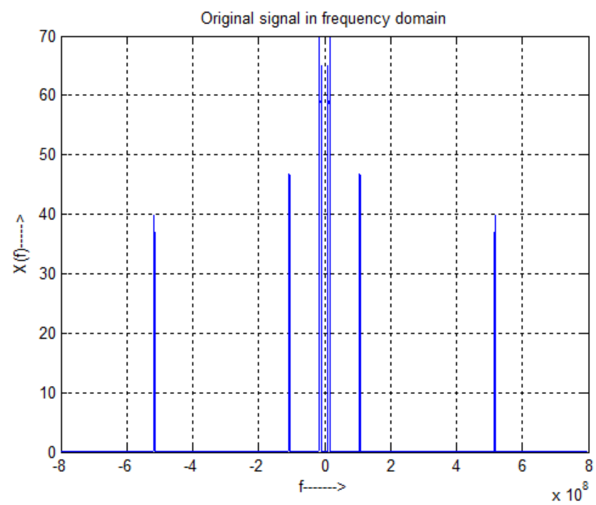

Fig. 4 Original Signal

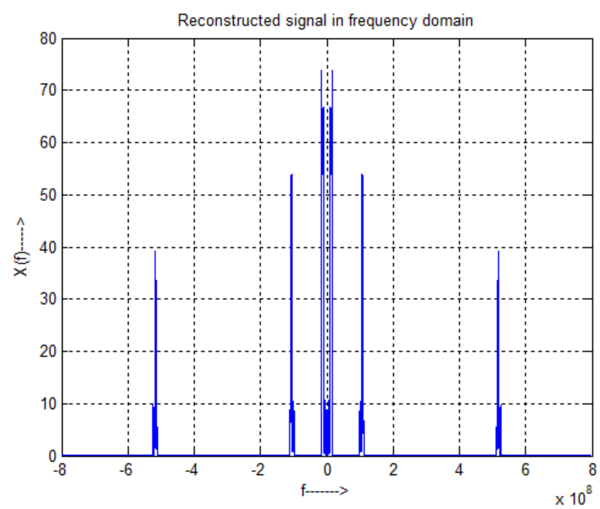

Fig. 5 Reconstructed signal

A sparse multiband signal with six active bands (including negative bands) was simulated in bandwidth range of 800Mhz. This is shown in Fig. 4.
This signal was sampled using MWC model. It was reconstructed using OMP algorithm directly from MMV. This is shown in Fig. 5. We can see from Fig. 4 and Fig. 5 that the

active bands have been recovered. By changing the number of channels in MWC model the recovery rate varies. This is shown in Fig. 6 and Fig. 7

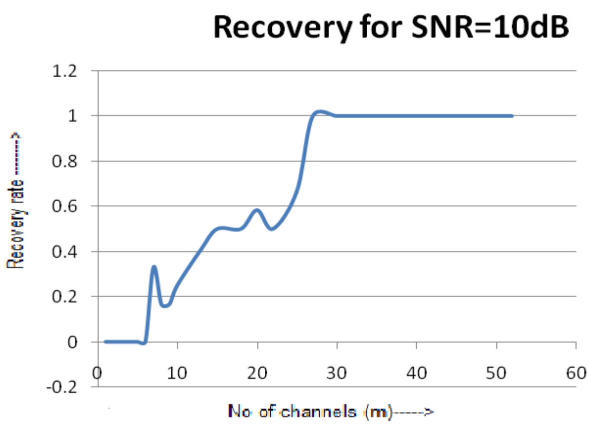

Fig. 6 Recovery Rate for SNR=10dB

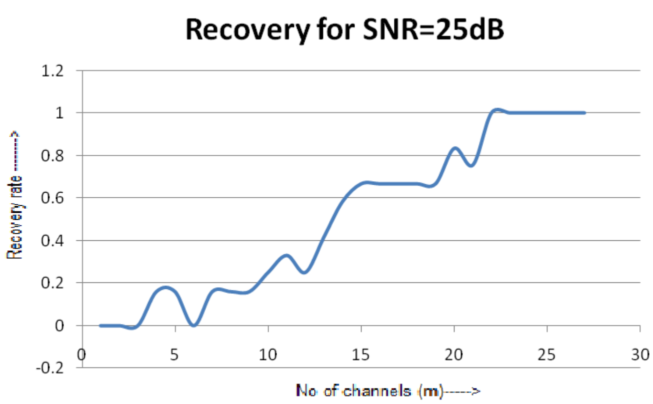

Fig. 7 Recovery Rate for SNR=25Db

5.3.2 Recovery After Conversion Of MMV To SMV

This algorithm deals with reduction of a MMV model into many SMV models. Consider a MMV model

$$Y = AX \qquad (2)$$

Where A is $m \times n$ rectangular matrix, Y is $m \times l$ matrix and X is $n \times l$ matrix. The matrix Y is converted into a single column matrix y by using a matrix a which follows some absolutely continuous distribution. The dimensions of a is $l \times 1$. Then, a SMV problem is solved to obtain the support. Once the support is known the signal X can be reconstructed. The Reduce MMV and Boost (ReMBo) algorithm is an iterative process. Input to this algorithm is K which denotes the sparsity required, iter which denotes the maximum number of allowed iterations, error E, P the probability distribution and S the optimization technique. The algorithm is as follows:

1. Take a vector 'a' of length 'l' that follows some probability distribution P.
2. Calculate $y = Ya$. This transforms Y into single dimensional vector 'y'. Solve $y = Ax$ using SMV technique.
3. Let support be $S = I(x)$.
4. If $S \le K$ then find X by inverting the relation in (1).
5. If $S > K$ then discard this S and go to step 2.
6. Perform this iterative process till the number of iterations is equal to iter.

By the end of this algorithm the spectral support S is found which may be directly used to find out the holes in the spectrum or the time domain signal can be reconstructed.

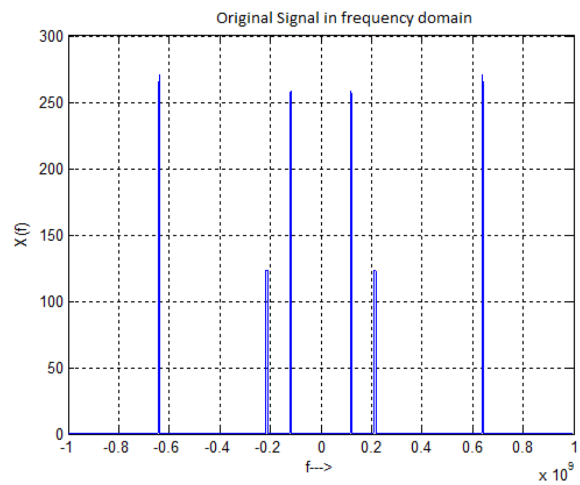

Fig. 8 Original Signal

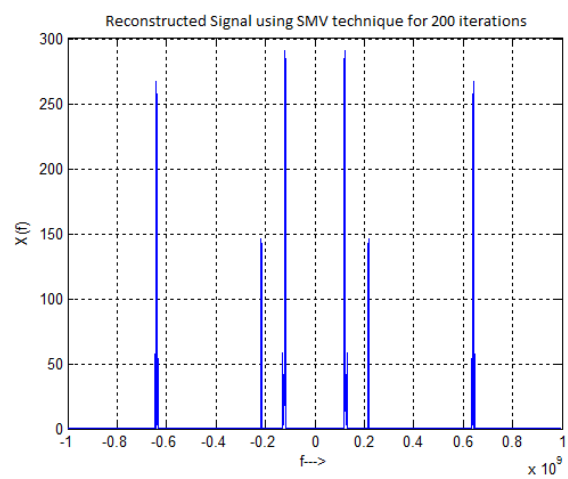

Fig. 9 Reconstructed signal for 200 iterations

A sparse multiband signal with six active bands (including negative bands) was simulated in bandwidth range of 800Mhz. This is shown in Fig 8.

This signal was sampled using MWC model. The MMV system is converted into SMV system using ReMBO algorithm [4] and then reconstructed using OMP algorithm for 200 iterations as shown in Fig. 9

Both methods discussed above show similar results except that the direct recovery from MMV involves less number of iterations than SMV. For the specifications mentioned in section IV, in MMV system the number of unknowns per iterations is 3204. In the SMV technique the number of unknowns per iteration is 267.

5.4 CONCLUSION

The MWC method recovers the required signal by sampling at a rate lesser than Nyquist rate and provides better results compared to Random demodulator and MC techniques for sparse multiband signals. Direct recovery from MMV system uses lesser number of channels to recover than Recovery from SMV system, but the number of unknown per iteration is higher in the former.

5.5 FUTURE SCOPE

The matrix A used in this method is randomly generated to ensure perfect reconstruction. But currently studies are going on to use deterministic matrix instead of random matrix. This method finds wide applications in Software Defined Radio (SDR).

ACKNOWLEDGEMENT

We would like to thank Ram R, Niranjan Manoharan, Archana C K, Manjari Das C and Krishna Teja V for their support during various stages of this work.

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